Tonality and functional equivalence: A multi-level model for the cognition of triadic progressions in 19th century music

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The subject of this paper is the cognition of triadic progressions in 19th century tonal music. Music psychological research concerning the cognition of harmonic progressions mainly relies on diatonic music in which triads are easily relatable to a key. Triadic distance is therefore measured in terms of root relationships to the tonal center, the tonic (Krumhansl & Kessler, 1982). This conception is not directly applicable to chromatic music where musical coherence is not only obtained by common a key. Transformational music theory puts strong emphasis on voice-leading parsimony as a measure of distance. The most efficient transformations between major and minor triads are *P* (parallel), *R* (relative) and *L* (leading-tone exchange), which is also in accordance with empirical findings of diatonic triadic relatedness (Krumhansl, 1998). Notably, *P* and *R* generate an octatonic scale containing eight major and minor triads which are claimed to be *functionally equivalent*. Transformational analyses result in sequential patterns of triadic progressions and an overarching key is not required. Based on an extended notion of function and acknowledging that there are compelling arguments for both hierarchical and sequential representations of the cognition of harmonic progressions a multi-level model is proposed that combines both approaches, adopting features of the generative model by Rohrmeier (2011). The two main components of the model are the concept of functional equivalence and the distinction between the hierarchic-syntactic cognition of functional progressions and the schematic cognition of functional values.

The subject of this paper is the representation of the cognition of triadic progressions in 19th century tonal music. The proposed model is supported both by music theoretic literature (historic as well as contemporary) and music psychological research. Despite the fact that different harmonies are interrelated by distinct physical ratios of frequencies, tonality is a cognitive concept that cannot be found in the external musical stimuli but in the human mind. Cognitive models of tonality should reflect the fact that this concept evolved and changed its meaning over time and that it is possible for contemporary listeners to distinguish between different kinds of tonalities. In other words, "the perception of musical structure depends on the processing of pitch information with reference to a system of knowledge about the conventional uses of pitches within the musical tradition" (Krumhansl & Kessler, 1982, p. 334). This paper attempts to present such a model.

The first section provides the theoretical background from transformational music theory and introduces the concept of *functional equivalence*. The second section briefly summarizes important empirical research on tonality perception and reveals difficulties when applying the results to 19th century tonality. Special attention is given to the debate whether tonal music has syntactic features comparable to language or not. In the third and main section a cognitive multi-level model for triadic progressions is outlined. The model attempts to reconcile the syntax- vs. schema-based accounts in assuming that both cognitive processes take part in the cognition of 19th century harmony. It is also mentioned how this model can inspire further empirical research and open questions are addressed. A final short conclusion points out the main statements and results of this paper.

If not mentioned otherwise, uppercase letters (e.g., C, Db, F#) indicate major triads and lowercase letters (e.g., c, db, f#) indicate minor triads. Italic abbreviations (e.g., *P*, *auth*, *Id*) stand for transformations.

A Riemannian motivation

The chord sequence in Fig. 1 is presented by Hugo Riemann in the final chapter on tonality

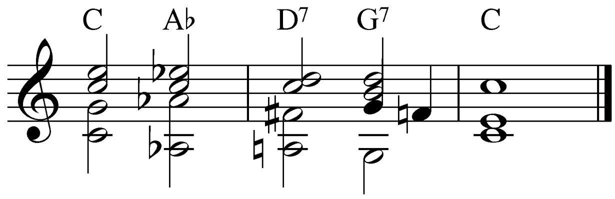

Figure 1. Chord sequence presented by Riemann (1880).

of his "Outline of a new method of harmonic theory"[1] (Riemann, 1880, p. 67). This sequence will be employed as a representative for triadic progressions in 19th century tonality. Riemann observes:

"The *old conception of key*, meaning a *harmony based on [diatonic] scales* is by now faltering because of the 'altered chords'; the categories of altered chords in the old sense of 'chords with tones not belonging to a scale' had to be extended too disproportionately, if one wanted to hold onto the scale-key, facing today's liberal harmony. Progressions like [Fig. 1] mock any old-fashioned schematization; even if one would ridiculously employ three or four different keys to clarify the matter, it wouldn't fit" (Riemann, 1880, p. 67, highlighting original, translation by the author).

Subsequently, an analysis in terms of key scale-degrees is an inadequate attempt and other explications are needed[2].

Music theory

Transformational music theory

Being a subdiscipline of mathematical music theory, transformational theory[3] uses mathematical concepts for musical analysis, especially from group theory[4]. Strong emphasis is put on voice-leading parsimony between triads or other pitch collections as a measure of distance (Cohn, 1997; Douthett & Steinbach, 1998; Lewin, 1980). The smaller the amount of voice-leading steps, the closer the relationship. Thus, the most efficient transformations between major and minor triads regarding parsimony are those that raise or lower only a single voice by a tone or a semitone. These are the relative transformation (*R*) that raises the fifth of a major triad and lowers the root of a minor triad by a full step; the parallel transformation (*P*) that lowers or raises the third of a major or minor triad by a half step, respectively; and the leading-tone transformation (*L*) that lowers the root of a major triad and raises the fifth of a minor triad by a semitone. It turns out that these voice-leadings coincide with inversions of the triad at one of its constituting intervals. The left side of Fig. 2 shows those inversions of C. The black solid lines visualize the inverted interval. *P* inverts the fifth, *R* inverts the major third and *L* inverts the minor third. The right side of Fig. 2 shows the triads embedded into

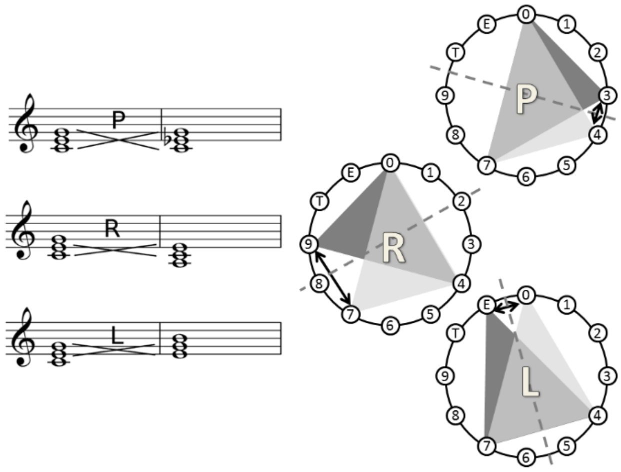

Figure 2. Effects of the transformations *P*, *R* and *L* on C {0, 4, 7} (light grey). The dashed line marks the mirror axis, the arrows mark parsimonious voice-leading; T and E are abbreviations for 10 and 11, respectively.

the pitch-class circle. The black double-headed arrows show the voice-leading and the grey dashed lines visualize the corresponding inversional mirror axis[5].

The dominant transformation *D* that moves a triad by a fifth downwards is not parsimonious but musically of great importance. The identity transformation *Id* doesn't change anything but is necessary for formal reasons. Transformational analyses often employ an abstract map to depict tonal relations, the "Tonnetz" (tone net, Fig. 3), most commonly spanned between a fifth axis (horizontal) and a major third axis (lower left to upper right).

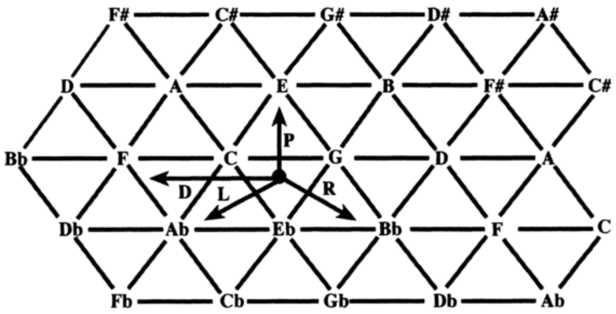

Figure 3. The "Tonnetz", reprinted from Cohn (1998).

Through concatenation[6] of the triadic transformations it is possible to "navigate" on the Tonnetz through paths like on a map. An overarching key to which each member of the sequence can be related is not required. This fact predestines this method for the analysis of 19th century music where a key is sometimes difficult to identify. The transformational analysis of the chord progression C–Ab–D–G–C is shown in Fig. 4[7]. The initial C major triad is marked with an asterisk (*).

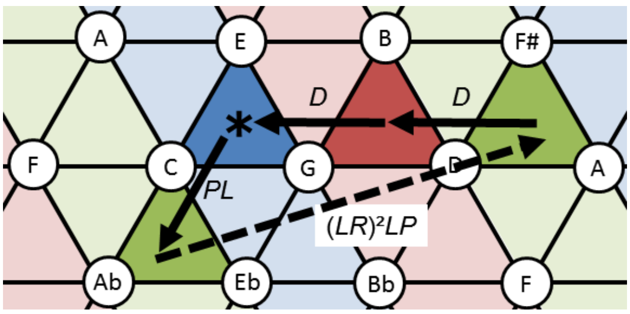

Figure 4. Transformational analysis of the triadic sequence C–Ab–D–G–C. The asterisk (*) marks the initial and final triad C.

Function and functional equivalence

The *function* of a chord in diatonic contexts is usually identified with its root's position in the scale of a given key. But according to Hugo Riemann (1880, 1893) different chords can have the same function or, at least, substitute another functionally. Traditionally, *P* and *R* related chords are understood as being functionally very close[8]. Applying *P* and *R* alternately on a triad generates an octatonic scale containing eight major and minor triads (see Fig. 5) that corresponds to a diagonal path on the Tonnetz from upper left to lower right (colored differently in Fig. 4).

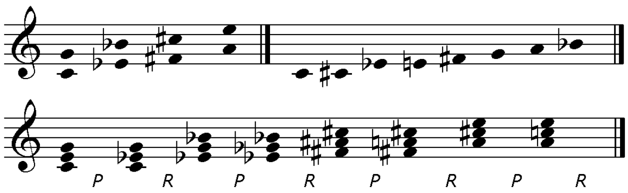

Figure 5. A function decomposed as minor third related fifths, as a scale and as a succession of *P* and *R* related triads.

According to music theorists of the Hungarian tradition (B. Haas, 2004; Lendvai, 1971, 1995; Schild, 2010) triads on this path share the same function, they are *functionally equivalent*[9]. Following this theoretical paradigm, "function" subsequently denotes the membership to the octatonic scale as well as its manifold triadic instantiations. Fig. 5 shows a function, decomposed into minor third separated fifths (top left), an octatonic scale (top right) and eight major and minor triads (bottom). There are three transpositions of a function which can in a concrete musical context be identified as tonic (T), dominant (D) and subdominant (S)[10]. This guarantees that each of the 24 minor or major triads can be uniquely assigned to a function.

In Fig. 4 and 6 the three functions are colored blue, red and green, respectively. The interaction of the three functions (their mutually shared pitch classes) is depicted in Fig. 6. There are two *functional progressions*: authentic and plagal, denoted *auth* and *plag*[11]. They are indicated by arrows on the left and the right of Fig. 6. Maintaining the same function is called *functional prolongation*[12], denoted *prol*, which is not shown in Fig. 6.

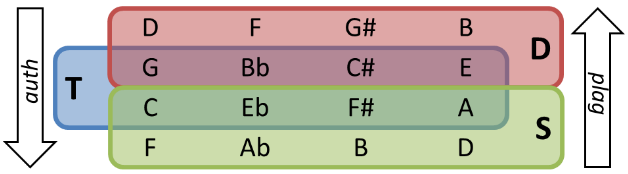

Figure 6. Interaction of the three functions tonic (T), dominant (D) and subdominant (S). Uppercase letters stand for pitch classes. The arrows on the left and right indicate functional progressions (*auth* and *plag*).

Empirical research on tonality

Probe-tone ratings and key-profiles

Empirical research on tonality attempts to find out which cognitive processes underlie the perception of harmony in tonal music[13]. One of the most influential publications on tonality perception was the study of Krumhansl and Kessler (1982). By asking listeners to rate how well a probe-tone fits into a given harmonic context (established by previous listening to different types of cadences) they deduced stability values for each member of the chromatic scale with ref-

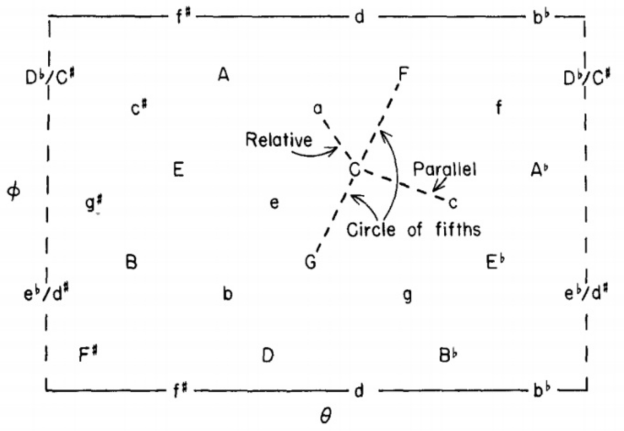

Figure 7. The representation of key distances obtained with multidimensional scaling, reprinted from Krumhansl & Kessler (1982). The letters stand for major and minor keys.

erence to the tonal center. Those values were used to generate key-profiles for the major and minor mode, and a four-dimensional map of interkey distances was produced using multidimensional scaling (Fig. 7). On this map perceived distances are represented by geometric distances. As can be seen the closest perceived relationships between keys are in astonishing accordance with parsimonious voice-leadings between triads. This means, Krumhansl and Kessler's representation is qualitatively equivalent to the Tonnetz[14]. Both depictions have the shape of a torus, meaning that opposing sides coincide.

The next step was to measure a chord's function by determining the distance of its root in reference to the tonal center of a key. Not very surprisingly, it turned out that triads are perceived most closely to those keys in which they play significant functional roles[15], emphasizing again the prominence of the parsimonious transformations.

This study has inspired a great deal of subsequent research on tonality (e.g. Bharucha & Krumhansl, 1983; Temperley, 2001) though remarkably there have been no serious attempts to leave the key-bound diatonic context so far. Even if chromatic music (i.e. *not* diatonic music) is considered in research either diatonic scales are the point of departure or any tonal context is forsaken (e.g. atonal music) (Woolhouse & Cross, 2010; Woolhouse, 2012).

However, Riemann's cadence (Fig. 1) seems to challenge the tonal hierarchies found by Krumhansl and Kessler. Ab is rather distant from the C-major key and Ab and D are even further apart (see Fig. 7)[16]. Also recall that a transformational analysis requested only a few steps to generate this sequence[17] and no modulation occurs. The overall impression is that of a tonally coherent cadence. Considering that empirical research strongly points to a hierarchic structure of tonal harmony but acknowledging that music theory suggests sequential analyses of 19th century triadic phrases, there is a gap that needs to be bridged.

Tonal syntax or harmonic schemata?

Syntactic models of tonality trace back at least to the groundbreaking publication of "A Generative Theory of Tonal Music" (Lerdahl & Jackendoff, 1983) that incorporates linguistic conceptions and presumes that music's syntactic features are comparable, if not essentially identical, to linguistic ones. It has become evident that a great deal of empirical findings account for a hierarchical representation of harmonic structure[18]. More recently, there has also been support from neuroscience for the existence of musical hierarchies (Fitch & Martins, 2014; Koelsch, Rohrmeier, Torrecuso, & Jentschke, 2013; Patel, 2008, 2012).

Critics of a parallelism between language and music state that "tones and chords are not really syntactic at all" (London, 2012, p. 242) and that musical function is better understood in terms of *schemata*. Schemata model "how

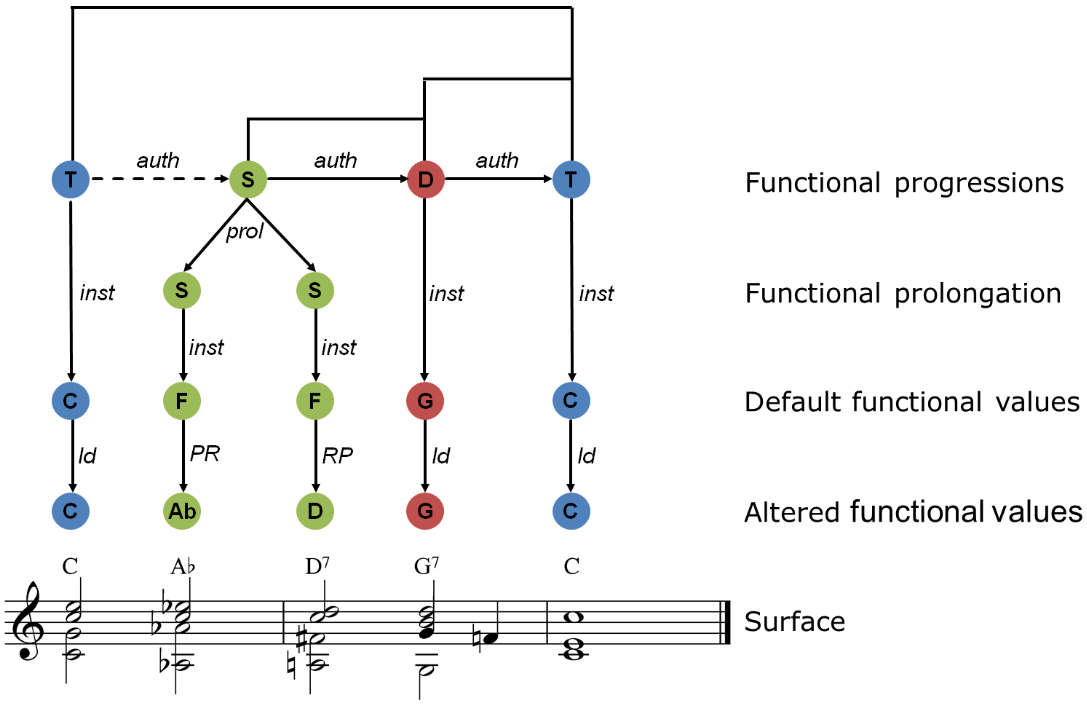

Figure 8. The multi-level model. On top, hierarchic functional progression is given by *auth*, prolongation by *prol*. Default values are assigned by *inst*. The default value for S is altered by *PR* and *RP*, respectively. The resulting triadic progression reflects the musical surface.

humans organize experience into structured categories. [They have] particular variable and default values, and can be intricately organized in ways unique to the domain" (Lerdahl, 1991, p. 273). This was already suggested by Krumhansl and Kessler:

"Both the construction and perception of pitch sequences around one or a few tones may reflect a general cognitive phenomenon that has been shown to apply to a wide variety of perceptual and semantic domains [...]. According to this view there are certain members of natural categories that function as cognitive reference points, or prototypes, for the category as a whole. These elements are described as the most representative of the categories, in relation to which all other category members are seen" (Krumhansl & Kessler, 1982, p. 363).

Despite the fact that hierarchic and schematic cognition are two distinct cognitive mechanisms, the present approach suggests that they are tightly intertwined and may both underlie the cognition of triadic progressions of 19th century music.

A multi-level model

Based on the extended notion of function that assumes functional equivalence for triads contained in an octatonic scale, and having in mind that there are compelling arguments for both hierarchical and schematic representations of harmonic progressions a multi-level approach is proposed.

Hierarchic functional progressions and schematic functional values

The main components of the model are the concept of functional equivalence and the distinction between the hierarchic-syntactic cognition of functional harmonic progressions and the schematic cognition of tonal functional values, following Rohrmeier (2007, 2011)[19]. He distinguishes between a functional and a scale degree level[20], tearing apart functionality and diatonic root position. This is worth mentioning because a great deal of empirical research on tonality fails to make this important differentiation. The psychological independence of the dominant transformation and the parsimonious transformations (Krumhansl, 1998) might also support the view that they are perceived on a structurally different level.

Functional progressions are modelled as hierarchic branchings (Lerdahl & Jackendoff, 1983; Lerdahl, 1991) and are denoted by *auth* and *plag*. Applying *auth* corresponds to the highest branching off a predefined T function, while *plag* corresponds to the highest branching off a D function. Functional prolongation is given by *prol* which expands a function on the hierarchic level. Assigning the functional default value is called instantiation[21] (denoted by *inst*). It serves as a mediator between the hierarchical level of functional progressions and the schematic level of functional values. The alteration of a functional value is given by *P*, *R* and any of their mutual concatenations.

An example

Fig. 8 shows the multi-level model with reference to Riemann's cadence (Fig. 1). The overall functional progressions are modelled hierarchically revealing an authentic T–S–D–T cadence. On the next level the subdominant function (S) is expanded by *prol*. Then, the functional default values are instantiated by *inst*. Finally, the default value of the subdominant (F) is altered via *PR* and *RP* into Ab and D, respectively, so that Riemann's cadence results as the musical surface.

Discussion and open questions

Obviously the multi-level model is to date not elaborated entirely. Very important for the further development will be its formalization in a unified way. The example above describes a top-down process. The model can also be understood as a bottom-up representation where starting from the musical surface the model unfolds step by step until a hierarchic functional understanding is achieved. To clarify if a top-down or a bottom-up representation is more appropriate will be the scope of further research.

Another issue concerns further empirical research. The concept of functional equivalence is theoretically convincing but not yet verified. Does the octatonic scale provide an adequate context to generate a stable feeling of tonality for listeners in an experiment? Can *functional profiles* be generated using the probe-tone method? This might be difficult because it can be assumed that the diatonic scale and the related feeling of a tonal center of a key is anchored much more deeply in most listeners than the sense for functional equivalence in chromatic romantic music. The high degree of symmetry of the octatonic scale itself prevents the existence a tonal center (Messiaen, 1944). Admittedly some tones can be understood as structurally more important, because they form the basis of a fifth interval (see first bar of Fig. 5) and the fifth is an interval both music theoretically and psychoacoustically understood as fundamental (Handschin, 1948; von Helmholtz, 1863) but it is uncertain if this suffices to establish a stable tonal context as claimed by music theory (B. Haas, 2004; Lendvai, 1971, 1995; Messiaen, 1944; Schild, 2010). As Krumhansl and Kessler note, "the hierarchy of tonal stability is acquired through experience with the structural relations that obtain in the music itself" (Krumhansl & Kessler, 1982, p. 364). It can be assumed that intense exposure to chromatic music as well as an apt system of concepts (being both concise and unequivocal) can contribute to create a solid perception of functional equivalence.

Also, because both theoretical and empirical scholars prefer a toroidal representation of tonal space, one could wonder if this torpedoes the concept of hierarchy because a torus has no fixed center whereas a hierarchy needs a highest element. This affects particularly the initial progression in Fig. 1 from C to Ab. While C is the highest element of the hierarchy, functional equivalence implies that it is also subordinated to Ab because it is an authentic progression. This contradiction is shown by the dashed arrow.

Conclusion

While I do not fully agree with the statement that "sequences of tones or chords are not really syntactic at all" (London, 2012, p. 242), I argue that, while there is an overall hierarchical understanding of harmonic progressions (and thus also of musical form), harmonic functions themselves can be viewed as schematic entities that have certain default values. These can be substituted with functionally equivalent chords, achieved via the application of the parsimonious *P* and *R* transformation.

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[1] "Skizze einer neuen Methode der Harmonielehre".

[2] It has to be emphasized that the present approach differs from Riemann's resolution. Nonetheless, his argument that traditional key-related root analysis is incapable to account for this harmonic progression still holds.

[3] Scholars that focus on the analysis of tonal music sometimes prefer the term "Neo-Riemannian Theory", honoring Hugo Riemann.

[4] A group is a set together with a binary operation that fulfills distinct features. The exact definition is not important here and can be looked up in any textbook of higher algebra. Important for the current discussion is that groups can be used to represent symmetric structures. For a short introduction, see Crans, Fiore, & Satyendra (2009).

[5] Subsequently, it will not be distinguished between voice-leading and inversional transformations, because we consider triads as pitch class sets.

[6] The order of the transformations reflects the order of their application, e.g. *PL* means "first *P*, then *L*".

[7] This is the same sequence as presented by Riemann (Fig. 1) except that the sevenths of D^7 and G^7 are omitted. The minor seventh of a chord is understood to reinforce the dominant feeling of that chord. This fact comes from contrapuntal rules where the seventh of a dominant is supposed to resolve into the major or minor third of a following tonic.

[8] *P* related keys share the same tonic, *R* related keys share the same scale.

[9] Functional equivalence does not imply that equivalent triads are indistinguishable.

[10] The cognitive process of tonic-finding is a research area on its own and cannot be reproduced here.

[11] These names are adopted from music theory where "authentic" (or "perfect") and "plagal" describe cadences that employ falling or rising fifths, respectively.

[12] Recall that the the dominant (*D*) and the identity transformation (*Id*) were defined for triads. Functional progressions (*plag* and *auth*) and functional prolongation (*prol*) refer to functions (Lerdahl & Jackendoff, 1983; Lerdahl, 2001).

[13] The question of tonality induction, how one learns structural features of tonality (Rohrmeier & Cross, 2009; Rohrmeier & Rebuschat, 2012; Tillmann, Bharucha, & Bigand, 2000; Vos, 2000) is not addressed here.

[14] The *L* relation is not shown in Fig. 6.

[15] This is a different concept of function as the Riemannian one in the present context.

[16] Fig. 6 represents perceived distances of keys not chords. Nonetheless, a chord is most closely perceived to the key in which it is the tonic. We thus assume that Ab and D are the closest to the respective keys.

[17] The shortest way from Ab to D is $(RP)^2=(LR)^2LP$. For graphical reasons it is not shown in Fig. 4.

[18] Krumhansl and Kessler recognized that "a hierarchy of levels is believed to exist in the instantiation of tonal regions" (1982, p. 336).

[19] Although Rohrmeier's rule-based model explicitly deals with diatonic music, it facilitates the extension of its application to chromatic music because it understands tonal harmony as a context-free grammar. See also W. B. de Haas (W. B. de Haas, 2012).

[20] In fact, he distinguishes between four levels: phrase level, functional level, scale degree level and surface level. Here, only the distinction between functional and scale degree level is taken into account.

[21] Rohrmeier speaks of "the generation of elementary functional chord terms from functional regions" (Rohrmeier, 2011, p. 40). This is primarily a formal assumption, because every schema is believed to have a default value. In an abstract context it might be likely that the default values of a function are unambiguously determined, but in a concrete musical context many factors might easily interfere.